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Some Intuitionistic Topological Notions of Intuitionistic Region, Possible Application to GIS Topological Rules

A. A. Salama[1], Mohamed Abdelfattah[2], S. A. Alblowi[3]

[1]Department of Mathematics and Computer Science, Faculty of Sciences, Port Said University, Egypt
[2]Information System Department, Faculty of Computers & Information, Benha University, EGYPT
[2*]Information System Department, Faculty of Computers & Information, Islamic University, KSA
[3]Department of Mathematics, King Abdulaziz University, Jeddah, KSA

Abstract: In Geographical information systems (GIS) there is a need to model spatial regions with intuitionistic boundary. In this paper, we generalize the topological ideals spaces to the notion of intuitionistic set; we construct the basic fundamental concepts and properties of an intuitionistic spatial region. In addition, we introduce the notion of ideals on intuitionistic set which is considered as a generalization of ideals studies in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. The important topological intuitionistic ideal has been given. The concept of intuitionistic local function is also introduced for a intuitionistic topological space. These concepts are discussed with a view to find new intuitionistic topology from the original one. The basic structure, especially a basis for such generated intuitionistic topologies and several relations between different topological intuitionistic ideals are also studied here. Possible application to GIS topology rules are touched upon.

Keywords: Intuitionistic Set, Intuitionistic Ideal, Intuitionistic Topology; Intuitionistic Local Function; Intuitionistic Spatial Region; GIS.

1. INTRODUCTION

In Geographical information systems (GIS) there is a need to model spatial regions with intuitionistic boundary. Ideal is one of the most important notions in general topology. A lot of different kinds of ideals have been introduced and studied by many topologists [1-16]. Throughout a few last year's many types of sets via ideals have been defined and studied by a staff of topologists. As a result of these new sorts of sets, topologists used some of them to construct new forms of topological spaces. This helps us to present several types of functions and investigate some operators which join between the above constructed spaces. In this paper, we generalize the topological ideals spaces to the notion of intuitionistic set; we construct the basic concepts of the intuitionistic topology. In addition, we introduce the notion of ideals on intuitionistic set which is considered as a generalization of ideals studies in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. The important topological intuitionistic ideal has been given. The concept of intuitionistic local function is also introduced for a intuitionistic topological space. These concepts are discussed with a view to find new intuitionistic topology from the original one. The basic structure, especially a basis for such generated intuitionistic topologies and several relations between different topological intuitionistic ideals are also studied here.

2. PRELIMINARIES

We recollect some relevant basic preliminaries, and in particular, the work of Hamlett, Jankovic and Kuratowski et al. in [4, 5, 6, 7, 8, 10, 11, 12], Abd El-Monsef et al.[1, 2, 3] and Salama et al. [13, 14]

3. SOME INTUITIONISTIC TOPOLOGICAL NOTIONS OF INTUITIONISTIC REGION

Here we extend the concepts of sets and topological space to the case of intuitionistic sets.

Definition 3.1 : Let X be a non-empty fixed set. A intuitionistic set(IS for short) A is an object having the form

$A=\langle A_1, A_2\rangle$ where A_1, A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The intuitionistic empty set is $\phi_I = \langle \phi, X\rangle$ and the intuitionistic universal set is $X_I = \langle X, \phi\rangle$.

Here we extend the concepts of topological space to the case of intuitionistic sets.

Definition 3.2: An intuitionistic topology (IT for short) on a non-empty set X is a family Γ of intuitionistic subsets in X satisfying the following axioms

i) $\phi_I, X_I \in \Gamma$.

ii) $A_1 \cap A_2 \in \Gamma$ for any A_1 and $A_2 \in \Gamma$.

iii) $\cup A_j \in \Gamma \ \forall \ \{A_j : j \in J\} \subseteq \Gamma$.

In this case the pair (X, Γ) is called a intuitionistic topological space $(ITS$ for short) in X. The elements in Γ are called intuitionistic open sets (IOSs for short) in X. An intuitionistic set F is closed if and only if its complement F^C is an open intuitionistic set.

Remark 3.1 Intuitionistic topological spaces are very natural generalizations of topological spaces, and they allow more general functions to be members of topology.

Example 3.1 Let $X = \{a, b, c, d\}$, ϕ_I, X_I be any types of the universal and empty subsets, and A, B are two intuitionistic subsets on X defined by $A = \langle \{a\}, \{b, d\}\rangle$, $B = \langle \{a\}, \{b\}\rangle$, then the family $\Gamma = \{\phi_I, X_I, A, B\}$ is a intuitionistic topology on X.

Definition 3.3

Let $(X, \Gamma_1), (X, \Gamma_2)$ are two intuitionistic topological spaces on X. Then Γ_1 is said be contained in Γ_2 (in symbols $\Gamma_1 \subseteq \Gamma_2$) if $G \in \Gamma_2$ for each $G \in \Gamma_1$. In this case, we also say that Γ_1 is coarser than Γ_2.

Proposition 3.1

Let $\{\Gamma_j : j \in J\}$ be a family of ITs on X. Then $\cap \Gamma_j$ is a intuitionistic topology on X. Furthermore, $\cap \Gamma_j$ is the coarsest IT on X containing all topologies

Proof:
Obvious

Now, we define the intuitionistic closure and intuitionistic interior operations on intuitionistic topological spaces:

Definition 3.4

Let (X, Γ) be ITS and $A = \langle A_1, A_2\rangle$ be a IS in X. Then the intuitionistic closure of A (ICl (A) for short) and intuitionistic interior (IInt (A) for short) of A are defined by

$$ICl(A) = \cap\{K : K \text{ is an IS in X and } A \subseteq K\}, \ IInt(A) = \cup\{G : G \text{ is an IOS in X and } G \subseteq A\},$$

where IS is a intuitionistic set, and IOS is a intuitionistic open set.

It can be also shown that $ICl(A)$ is a ICS (intuitionistic closed set) and $IInt(A)$ is a IOS in X

a) A is in X if and only if $ICl(A) \supseteq A$.

b) A is a ICS in X if and only if $IInt(A) = A$.

Proposition 3.2

For any intuitionistic set A in (X, Γ) we have

(a) $ICl(A^c) = (IInt(A))^c$,

(b) $IInt(A^c) = (ICl(A))^c$.

Proof:

a) Let $A = \langle A_1, A_2 \rangle$ and suppose that the family of intuitionistic subsets contained in A are indexed by the family if ISs contained in A are indexed by the family $A = \{< A_{j_1}, A_{j_2} >: i \in J\}$. Then we see that we have two types of $IInt(A) = \{< \cup A_{j_1}, \cup A_{j_2} >\}$ or $IInt(A) = \{< \cup A_{j_1}, \cap A_{j_2} >\}$ hence $(IInt(A))^c = \{< \cap A_{j_1}, \cap A_{j_2} >\}$ or $(IInt(A))^c = \{< \cap A_{j_1}, \cup A_{j_2} >\}$. Hence $ICl(A^c) = (IInt(A))^c$, which is analogous to (a).

Proposition 3.3

Let (X, Γ) be a ITS and A, B be two intuitionistic sets in X. Then the following properties hold:

(a) $IInt(A) \subseteq A$,
(b) $A \subseteq ICl(A)$,
(c) $A \subseteq B \Rightarrow IInt(A) \subseteq IInt(B)$,
(d) $A \subseteq B \Rightarrow ICl(A) \subseteq ICl(B)$,
(e) $IInt(A \cap B) = IInt(A) \cap IInt(B)$,
(f) $ICl(A \cup B) = ICl(A) \cup ICl(B)$,
(g) $IInt(X_I) = X_I$,
(h) $ICl(\phi_I) = \phi_I$

Proof. (a), (b) and (e) are obvious; (c) follows from (a) and from definitions.

Now, we add some further definitions and propositions for an intuitionistic topological region.

Corollary 3.1

Let $A = < A_{1}, A_2 >$ and $B = < B_1, B_2 >$ are two intuitionistic sets on a intuitionistic topological space (X, τ) then the following are holds

i) $I\operatorname{int}(A) \cap I\operatorname{int}(B) = I\operatorname{int}(A \cap B)$,
ii) $Icl(A) \cup Ncl(B) = I\operatorname{int}(A \cup B)$,
iii) $I\operatorname{int}(A) \subseteq A \subseteq Icl(A)$,
iv) $(I\operatorname{int}(A))^c = Icl(A^c)$, $(Icl(A))^c = I\operatorname{int}(A^c)$.

Definition 3.5

We define a intuitionistic boundary (NB) of a intuitionistic set $A = < A_{1}, A_2 >$ by: $I\partial A = Icl(A) \cap Icl(A^c)$.

The following theorem shows the intersection methods no longer guarantees a unique solution.

Corollary 3.2

$I\partial A \cap I\,\text{int}(A) = \phi_I$ iff $I\,\text{int}(A)$ is crisp (i.e., $I\,\text{int}(A) = \phi_I$ or $I\,\text{int}(A) = X_I$).

Proof :

Obvious

Definition 3.6

Let $A = < A_{1,}, A_2 >$ be a intuitionistic sets on a intuitionistic topological space (X, τ). Suppose that the family of intuitionistic open sets contained in A is indexed by the family $< A_{1,j}, A_{2j} >: j \in J$ and the family of intuitionistic open subsets containing A are indexed the family $< A_{1,ij}, A_{2i} >: i \in J$ .Then two intuitionistic interior, clouser and boundaries are defined as following

a) $I\,\text{int}(A)_{[\;]}$ defined as

$$I\,\text{int}(A)_{[\;]} = < \cup (A_{1j}), \cap (A_{2_j})^c >$$

b) $I\,\text{int}(A)_{<\,>}$ defined as

i) Type 1. $I\,\text{int}(A)_{<\,>} = < \cup (A_{1_i}), \cap (A_{2_i}) >$

c) $Icl(A)_{[\;]}$ may be defined as

$$Icl(A)_{[\;]} = < \cup (A_{1_j}), \cup (A_{2_i})^c >$$

d) $Icl(A)_{<\,>}$ defined as

$$Icl(A)_{<\,>} = < \cap (A_{2_i})^c, \cup (A_{2_1})^c >$$

e) Intuitionistic boundaries defined as

i) $I\partial A_{[\;]} = Icl(A_{[\;]}) \cap Icl(A^c{}_{[\;]})$

ii) $I\partial A_{<\,>} = Icl(A_{<\,>}) \cap Icl(A^c{}_{<\,>})$

Proposition 3.4

a) $I\,\text{int}(A)_{[\;]} \subseteq I\,\text{int}(A) \subseteq I\,\text{int}(A)_{<\,>}$,

b) $Icl(A)_{[\;]} \subseteq Icl(A)\; Icl(A)_{<\,>}$

c) $I\,\text{int}(A_{\{[\;],<\,>\}}) = \{[\;], <\,>\} I\,\text{int}(A)$ and $Icl(A_{\{[\;],<\,>\}}) = \{[\;], <\,>\} Icl(A)$

Proof:

We shall only prove (c), and the others are obvious.

$[\;]\, I\,\text{int}(A) = < \cup (A_{1_i}), (\cup A_{1_i})^c >$ Based on knowing that $(X_I - \cup A_{1_i}) = \cap (X_I - A_{1_i})$ then $[\;]\, I\,\text{int}(A) = < \cup (A_{1_i}), \cap (X_I - A_{1_i}) >$ In a similar way the others can prove.

Proposition 3.5

a) $I\operatorname{int}(A_{\{[\ ],<\ >\}}) = (I\operatorname{int}(A))_{\{[\ ],<\ >\}}$

b) $Icl(A_{\{[\ ],<\ >\}})_{\{[\ ],<\ >\}} = (Icl(A))_{\{[\ ],<\ >\}}$

Proof:
Obvious

Definition 3.6

Let $A=<A_1, A_2>$ be a intuitionistic sets on a intuitionistic topological space (X,τ). We define intuitionistic exterior of A as follows: $A^{IE} = X_I \cap A^C$

Definition 3.7

Let $A=<A_1, A_2>$ be a intuitionistic open sets and $B=<B_1, B_2>$ be a intuitionistic set on a intuitionistic topological space (X,τ) then

a) A is called intuitionistic regular open iff $A = I\operatorname{int}(Icl(A))$.

b) If $B \in IS(X)$ then B is called intuitionistic regular closed iff $A = Icl(I\operatorname{int}(A))$.

Now, we shall obtain a formal model for simple spatial intuitionistic region based on intuitionistic connectedness.

Definition 3.8

Let $A=<A_1, A_2>$ be a intuitionistic sets on a intuitionistic topological space (X,τ). Then A is called a simple intuitionistic region in connected NTS, such that

i) $Icl(A)$, $Icl(A)_{[\ ]}$, and $Icl(A)_{<\ >}$ are intuitionistic regular closed.

ii) $I\operatorname{int}(A)$, $I\operatorname{int}(A)_{[\ ]}$, and $I\operatorname{int}(A)_{<\ >}$ are intuitionistic regular open

iii) $I\partial(A)$, $I\partial(A)_{[\ ]}$, and $I\partial(A)_{<\ >}$ are intuitionistic connected.

Having $Icl(A)$, $Icl(A)_{[\ ]}$, $Icl(A)_{<\ >}$, $I\operatorname{int}(A)$, $I\operatorname{int}(A)_{[\ ]}$, $I\operatorname{int}(A)_{<\ >}$ are

$I\partial(A)$, $I\partial(A)_{[\ ]}$ and $I\partial(A)_{<\ >}$ for two intuitionistic regions, we enable to find relationships between two intuitionistic regions

4. INTUITIONISTIC IDEALS

Definition 4.1

Let X be non-empty set, and L a non–empty family of ISs. We call L a intuitionistic ideal (IL for short) on X if

i. $A \in L$ and $B \subseteq A \Rightarrow B \in L$ [heredity],

ii. $A \in L$ and $B \in L \Rightarrow A \cup B \in L$ [Finite additivity].

An intuitionistic ideal L is called a σ - intuitionistic ideal if $\{M_j\}_{j\in N} \subseteq L$, implies $\bigcup_{j\in J} M_j \in L$ (countable additivity).

The smallest and largest intuitionistic ideals on a non-empty set X are $\{\phi_I\}$ and the ISs on X. Also, IL_f, IL_c are denoting the intuitionistic ideals (IL for short) of intuitionistic subsets having finite and countable support of X respectively.

Moreover, if A is a nonempty IS in X, then $\{B \in IS : B \subseteq A\}$ is an IL on X. This is called the principal IL of all ISs, denoted by IL $\langle A \rangle$.

Remark 4.1

i. $\phi_{\mathrm{I}} \in L$

ii. If $X_{\mathrm{I}} \notin L$, then L is called intuitionistic proper ideal.

iii. If $X_{\mathrm{I}} \in L$, then L is called intuitionistic improper ideal.

Example 4.1

Let $X = \{a, b, c\}$, $A = \langle \{a\}, \{b, c\} \rangle$, $B = \langle \{a\}, \{c\} \rangle$, $C = \langle \{a\}, \{b\} \rangle$, $D = \langle \{a, b, c\}, \{c\} \rangle$, $E = \langle \{a, b\}, \{c\} \rangle$, $F = \langle \{a\}, \{a, c\} \rangle$, $G = \langle \{a\}, \{b, c\} \rangle$. Then the family $L = \{ \phi_{\mathrm{I}}, A, B, D, E, F, G \}$ of ISs is an IL on X.

Definition 4.2

Let L_1 and L_2 be two ILs on X. Then L_2 is said to be finer than L_1, or L_1 is coarser than L_2, if $L_1 \subseteq L_2$. If also $L_1 \neq L_2$. Then L_2 is said to be strictly finer than L_1, or L_1 is strictly coarser than L_2.

Two ILs said to be comparable, if one is finer than the other. The set of all ILs on X is ordered by the relation: L_1 is coarser than L_2; this relation is induced the inclusion in ISs.

The next Proposition is considered as one of the useful result in this sequel, whose proof is clear. $L_j = \langle A_{j_1}, A_{j_2} \rangle$.

Proposition 4.1

Let $\{L_j : j \in J\}$ be any non - empty family of intuitionistic ideals on a set X. Then $\bigcap_{j \in J} L_j$ and $\bigcup_{j \in J} L_j$ are intuitionistic ideals on X, where $\bigcap_{j \in J} L_j = \left\langle \bigcap_{j \in J} A_{j_1}, \bigcup_{j \in J} A_{j_2} \right\rangle$ or $\bigcap_{j \in J} L_j = \left\langle \bigcap_{j \in J} A_{j_1}, \bigcup_{j \in J} A_{j_2} \right\rangle$ and $\bigcup_{j \in J} L_j = \left\langle \bigcup_{j \in J} A_{j_1}, \bigcap_{j \in J} A_{j_2} \right\rangle$ or $\bigcup_{j \in J} L_j = \left\langle \bigcup_{j \in J} A_{j_1}, \bigcap_{j \in J} A_{j2} \right\rangle$.

In fact, L is the smallest upper bound of the sets of the L_j in the ordered set of all intuitionistic ideals on X.

Remark 4.2

The intuitionistic ideal defined by the single intuitionistic set ϕ_{I} is the smallest element of the ordered set of all intuitionistic ideals on X.

Proposition 4.2

A intuitionistic set $A = \langle A_1, A_2 \rangle$ in the intuitionistic ideal L on X is a base of L iff every member of L is contained in A.

Proof

(Necessity) Suppose A is a base of L. Then clearly every member of L is contained in A.
(Sufficiency) Suppose the necessary condition holds. Then the set of intuitionistic subsets in X contained in A coincides with L by the Definition 4.2.

Proposition 4.3

A intuitionistic ideal L_1, with base $A = \langle A_1, A_2 \rangle$, is finer than a intuitionistic ideal L_2 with base $B = \langle B_1, B_2 \rangle$, iff every member of B is contained in A.

Proof

Immediate consequence of the definitions.

Corollary 4.1

Two intuitionistic ideals bases A, B on X, are equivalent iff every member of A is contained in B and vice versa.

Theorem 4.1

Let $\eta = \langle A_{j_1}, A_{j_2} \rangle : j \in J$ be a non-empty collection of intuitionistic subsets of X. Then there exists a intuitionistic ideal $L(\eta) = \{A \in IS : A \subseteq \bigcup_{j \in J} A_j\}$ on X for some finite collection $\{A_j : j = 1, 2, \ldots, n \subseteq \eta\}$.

Proof

It's clear.

Remark 4.3

The intuitionistic ideal L (η) defined above is said to be generated by η and η is called sub-base of L (η).

Corollary 4.2

Let L_1 be an intuitionistic ideal on X and A $\in$ ISs, then there is an intuitionistic ideal L_2 which is finer than L_1 and such that A$\in$ L_2 iff $A \cup B \in L_2$ for each B$\in$ L_1.

Proof

It's clear.

Theorem 4.2

If $L = \{\phi_I, \langle A_1, A_2 \rangle\}$ is an intuitionistic ideals on X, then:

i) $[\ \ ]L = \{\phi_I, \langle A_1, A_2{}^c \rangle\}$ is an intuitionistic ideals on X.

ii) $\langle\ \ \rangle L = \{\phi_I, \langle A_2, A_1{}^c \rangle\}$ is an intuitionistic ideals on X.

Proof

Obvious

Theorem 4.3

Let $A = \langle A_1, A_2 \rangle \in L_1$, and $B = \langle B_1, B_2 \rangle \in L_2$, where L_1 and L_2 are intuitionistic ideals on X, then $A * B$ is an intuitionistic set $A * B = \langle A_1 * B_1, A_2 * B_2 \rangle$ where $A_1 * B_1 = \cup \{\langle A_1 \cap B_1, A_2 \cap B_2 \rangle\}$, $A_2 * B_2 = \cap \{\langle A_1 \cap B_1, A_2 \cap B_2 \rangle\}$.

5. INTUITIONISTIC POINTS AND NEIGHBOURHOODS SYSTEMS

Now we shall present some types of inclusions of a intuitionistic point and neighborhoods systems to a intuitionistic set:

Definition 5.1

Let $A = \langle A_1, A_2 \rangle$, be a intuitionistic set on a set X, then $p = \langle \{p_1\}, \{p_2\} \rangle$, $p_1 \neq p_2 \in X$ is called a intuitionistic point

An IP $p = \langle \{p_1\}, \{p_2\} \rangle$, is said to be belong to a intuitionistic set $A = \langle A_1, A_2 \rangle$, of X, denoted by $p \in A$.

Theorem 5.1

Let $A = \langle A_1, A_2 \rangle$, and $B = \langle B_1, B_2 \rangle$, be intuitionistic subsets of X. Then $A \subseteq B$ iff $p \in A$ implies $p \in B$ for any intuitionistic point p in X.

Proof

Clear

Theorem 5.2

Let $A = \langle A_1, A_2 \rangle$, be a intuitionistic subset of X. Then $A = \cup \{p : p \in A\}$.

Proof

Clear

Proposition 5.1

Let $\{A_j : j \in J\}$ is a family of ISs in X. Then

(a_1) $p = \langle\{p_1\},\{p_2\}\rangle \in \bigcap_{j\in J} A_j$ iff $p \in A_j$ for each $j \in J$.

(a_2) $p \in \bigcup_{j\in J} A_j$ iff $\exists j \in J$ such that $p \in A_j$.

.

Proposition 5.2

Let $A = \langle A_1, A_2\rangle$ and $B = \langle B_1, B_2\rangle$ be two intuitionistic sets in X. Then

a) $A \subseteq B$ iff for each p we have $p \in A \Leftrightarrow p \in B$ and for each p we have $p \in A \Rightarrow p \in B$.

b) $A = B$ iff for each p we have $p \in A \Rightarrow p \in B$ and for each p we have $p \in A \Leftrightarrow p \in B$.

Proposition 5.3

Let $A = \langle A_1, A_2\rangle$ be a intuitionistic set in X. Then $A = \cup < \{p_1 : p_1 \in A_1\},\{p_2 : p_2 \in A_2\}. >$.

Definition 5.3

Let $f : X \to Y$ be a function and p be a intuitionistic point in X. Then the image of p under f, denoted by $f(p)$, is defined by $f(p) = \langle\{q_1\},\{q_2\}\rangle$, where $q_1 = f(p_1), q_2 = f(p_2)$.

It is easy to see that $f(p)$ is indeed a IP in Y, namely $f(p) = q$, where $q = f(p)$, and it is exactly the same meaning of the image of a IP under the function f.

One can easily define a natural type of intuitionistic set in X, called "intuitionistic point" in X, corresponding to an element $p \in X$:

Definition 5.4

Let X be a nonempty set and $p \in X$. Then the intuitionistic point p_N defined by $p_N = \langle\{p\},\{p\}^c\rangle$ is called an intuitionistic point (IP for short) in X, where IP is a triple ({only one element in X}, the empty set,{the complement of the same element in X}).

Intuitionistic points in X can sometimes be inconvenient when expressing a intuitionistic set in X in terms of intuitionistic points. This situation will occur if $A = \langle A_1, A_2\rangle$, and $p \notin A_1$, where A_1, A_2 are three subsets such that $A_1 \cap A_2 = \phi$. Therefore we define the vanishing intuitionistic points as follows:

Definition 5.5

Let X be a nonempty set, and $p \in X$ a fixed element in X. Then the intuitionistic set $p_{N_N} = \langle\{p\},\{p\}^c\rangle$ is called "vanishing intuitionistic point" (VIP for short) in X, where VIP is a triple (the empty set,{only one element in X},{the complement of the same element in X}).

Example 5.1

Let $X = \{a,b,c,d\}$ and $p = b \in X$. Then $p_N = \langle\{b\},\{a,c,d\}\rangle$

Definition 5.6

Let $p_N = \langle\{p\},\{p\}^c\rangle$ be a IP in X and $A = \langle A_1, A_2\rangle$ a intuitionistic set in X.

(a) p_N is said to be contained in A ($p_N \in A$ for short) iff $p \in A_1$.

(b) Let p_{N_N} be a VIP in X, and $A=\langle A_1, A_2\rangle$ a intuitionistic set in X.
Then p_{N_N} is said to be contained in A ($p_{N_N} \in A$ for short) iff $p \notin A_2$.

Proposition 5.1

Let $\{D_j : j \in J\}$ is a family of ISs in X. Then

(a_1) $p_N \in \bigcap_{j\in J} D_j$ iff $p_N \in D_j$ for each $j \in J$.

(a_2) $p_{N_N} \in \bigcap_{j\in J} D_j$ iff $p_{N_N} \in D_j$ for each $j \in J$.

(b_1) $p_N \in \bigcup_{j\in J} D_j$ iff $\exists j \in J$ such that $p_N \in D_j$.

(b_2) $p_{N_N} \in \bigcap_{j\in J} D_j$ iff $\exists j \in J$ such that $p_{N_N} \in D_j$.

Proof
Straightforward.

Proposition 5.2

Let $A=\langle A_1, A_2\rangle$ and $B=\langle B_1, B_2\rangle$ be two intuitionistic sets in X. Then

c) $A \subseteq B$ iff for each p_N we have $p_N \in A \Leftrightarrow p_N \in B$ and for each p_{N_N} we have $p_N \in A \Rightarrow p_{N_N} \in B$.

d) $A = B$ iff for each p_N we have $p_N \in A \Rightarrow p_N \in B$ and for each p_{N_N} we have $p_{N_N} \in A \Leftrightarrow p_{N_N} \in B$.

Proof
Obvious.

Proposition 5.4

Let $A=\langle A_1, A_2\rangle$ be a intuitionistic set in X. Then
$A=(\cup\{p_N : p_N \in A\})\cup(\cup\{p_{NN} : p_{NN} \in A\})$.

Proof
It is sufficient to show the following equalities: $A_1 = (\cup\{p\ \} : p_N \in A\})\cup(\cup\{\phi : p_{NN} \in A\})$ and
$A_2 = (\cap\{\{p\ \}^c : p_N \in A\})\cap(\cap\{\{p\}^c : p_{NN} \in A\})$, which are fairly obvious.

Definition 5.7

Let $f : X \to Y$ be a function.

(a) Let p_N be a nutrosophic point in X. Then the image of p_N under f, denoted by $f(p_N)$, is defined by $f(p_N) = \langle \{q\}, \{q\}^c \rangle$, where $q = f(p\ )$.

(b) Let p_{NN} be a VIP in X. Then the image of p_{NN} under f, denoted by $f(p_{NN})$, is defined by $f(p_{NN}) = \langle \{q\}, \{q\}^c \rangle$, where $q = f(p\ )$.

It is easy to see that $f(p_N)$ is indeed a IP in Y, namely $f(p_N) = q_N$, where $q = f(p\ )$, and it is exactly the same meaning of the image of a IP under the function f.

$f(p_{NN})$ is also a VIP in Y, namely $f(p_{NN}) = q_{NN}$, where $q = f(p\ )$.

Proposition 5.4

Any IS A in X can be written in the form $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$, where $\underset{N}{A} = \cup\{p_N : p_N \in A\}$, $\underset{N}{A} = \phi_N$ and $\underset{NNN}{A} = \cup\{p_{NN} : p_{NN} \in A\}$. It is easy to show that, if $A = \langle A_1, A_2 \rangle$, then $\underset{N}{A} = \langle A_1, A_1^c \rangle$.

Proposition 5.5

Let $f : X \to Y$ be a function and $A = \langle A_1, A_2 \rangle$ be a intuitionistic set in X. Then we have $f(A) = f(\underset{N}{A}) \cup f(\underset{NN}{A}) \cup f(\underset{NNN}{A})$.

Proof

This is obvious from $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$.

Definition 5.8

Let p be a intuitionistic point of an intuitionistic topological space (X, τ). A intuitionistic neighbourhood (INBD for short) of a intuitionistic point p if there is a intuitionistic open set(IOS for short) B in X such that $p \in B \subseteq A$.

Theorem 5.1

Let (X, τ) be a intuitionistic topological space (ITS for short) of X. Then the intuitionistic set A of X is IOS iff A is a INBD of p for every intuitionistic set $p \in A$.

Proof

Let A be IOS of X . Clearly A is a INBD of any $p \in A$. Conversely, let $p \in A$. Since A is a IBD of p, there is a IOS B in X such that $p \in B \subseteq A$. So we have $A = \cup\{p : p \in A\} \subseteq \cup\{B : p \in A\} \subseteq A$ and hence $A = \cup\{B : p \in A\}$. Since each B is IOS.

6. INTUITIONISTIC LOCAL FUNCTIONS

Definition 6.1

Let (X, τ) be a intuitionistic topological spaces (ITS for short) and L be intuitionistic ideal (IL, for short) on X. Let A be any IS of X. Then the intuitionistic local function $IA^*(L, \tau)$ of A is the union of all intuitionistic points $P = \langle \{p_1\}, \{p_2\} \rangle$, such that if $U \in IN((p))$ and $IA^*(L, \tau) = \cup\{p \in X : A \cap U \notin L$ for every Unbd of IN(P)$\}$, $IA^*(L, \tau)$ is called a intuitionistic local function of A with respect to τ and L which it will be denoted by $NCA^*(L, \tau)$, or simply $IA^*(\mathrm{L})$.

Example 6.1

One may easily verify that.

If L=$\{\phi_I\}$, then $\mathrm{IA}^*(L, \tau) = Icl(A)$, for any intuitionistic set $A \in ISs$ on X.

If $\mathrm{L} = \{\text{all ISs on X}\}$ then $\mathrm{IA}^*(L, \tau) = \phi_I$, for any $A \in ISs$ on X .

Theorem 6.1

Let (X, τ) be a ITS and L_1, L_2 be two topological intuitionistic ideals on X. Then for any intuitionistic sets A, B of X. then the following statements are verified

i) $A \subseteq B \Rightarrow IA^*(L,\tau) \subseteq IB^*(L,\tau)$,
ii) $L_1 \subseteq L_2 \Rightarrow IA^*(L_2,\tau) \subseteq IA^*(L_1,\tau)$.
iii) $IA^* = Icl(A^*) \subseteq Icl(A)$.
iv) $IA^{**} \subseteq IA^*$.
v) $I(A \cup B)^* = IA^* \cup IB^*$.,
vi) $I(A \cap B)^*(L) \subseteq IA^*(L) \cap IB^*(L)$.
vii) $\ell \in L \Rightarrow I(A \cup \ell)^* = IA^*$.
viii) $IA^*(L,\tau)$ is an intuitionistic closed set .

Proof

i) Since $A \subseteq B$, let $p = \langle \{p_1\},\{p_2\} \rangle \in IA^*(L_1)$ then $A \cap U \notin L$ for every $U \in IN(p)$. By hypothesis we get $B \cap U \notin L$, then $p = \langle \{p_1\},\{p_2\} \rangle \in IB^*(L_1)$.

ii) Clearly. $L_1 \subseteq L_2$ implies $IA^*(L_2,\tau) \subseteq IA^*(L_1,\tau)$ as there may be other IFSs which belong to L_2 so that for GIFP $p = \langle \{p_1\},\{p_2\} \rangle \in IA^*(L_1)$ but P may not be contained in $IA^*(L_2)$.

iii) Since $\{\phi_I\} \subseteq L$ for any IL on X, therefore by (ii) and Example 3.1, $IA^*(L) \subseteq IA^*(\{0_I\}) = Icl(A)$ for any IS A on X. Suppose $P_1 = \langle \{p_1\},\{p_2\} \rangle \in Icl(A^*(L_1))$. So for every $U \in IN(P_1)$, $I(A^*) \cap U \neq \phi_I$, there exists $P_2 = \langle \{q_1\},\{q_2\} \rangle \in IA^*(L_1) \cap U$ such that for every V INBD of $P_2 \in N(P_2)$, $A \cap U \notin L$. Since $U \cap V \in IN(p_2)$ then $A \cap (U \cap V) \notin L$ which leads to $A \cap U \notin L$, for every $U \in N(P_1)$ therefore $P_1 \in I(A^*(L))$ and so $Icl(INA^*) \subseteq IA^*$ While, the other inclusion follows directly. Hence $IA^* = Icl(IA^*)$ .But the inequality $IA^* \subseteq Il(IA^*)$.

iv) The inclusion $IA^* \cup IB^* \subseteq I(A \cup B)^*$ follows directly by (i). To show the other implication, let $p \in I(A \cup B)^*$ then for every $U \in I(p)$, $(A \cup B) \cap U \notin L, i.e, (A \cap U) \cup (B \cap U) \notin L$. then, we have two cases $A \cap U \notin L$ and $B \cap U \in L$ or the converse, this means that exist $U_1, U_2 \in IN(P)$ such that $A \cap U_1 \notin L$, $B \cap U_1 \notin L$, $A \cap U_2 \notin L$ and $B \cap U_2 \notin L$. Then $A \cap (U_1 \cap U_2) \in L$ and $B \cap (U_1 \cap U_2) \in L$ this gives $(A \cup B) \cap (U_1 \cap U_2) \in L$, $U_1 \cap U_2 \in I(N(P))$ which contradicts the hypothesis. Hence the equality holds in various cases.

vi) By (iii), we have $IA^{*^*} = Icl(IA^*)^* \subseteq Icl(IA^*) = IA^*$

Let (X,τ) be a ITS and L be IL on X . Let us define the intuitionistic closure operator $Icl^*(A) = A \cup I(A^*)$ for any IS A of X. Clearly, let $Icl^*(A)$ is a intuitionistic operator. Let $I\tau^*(L)$ be IT generated by Icl^* .i.e $I\tau^*(L) = \{A : Icl^*(A^c) = A^c\}$. now $L = \{\phi_I\} \Rightarrow Icl^*(A) = A \cup IA^* = A \cup Icl(A)$ for every intuitionistic set A. So, $I\tau^*(\{\phi_I\}) = \tau$. Again $L = \{all \text{ ISs on X}\} \Longrightarrow Icl^*(A) = A,$ because $IA^* = \phi_I$, for every intuitionistic set A so $I\tau^*(L)$ is the intuitionistic discrete topology on X. So we can coIlude by Theorem 4.1.(ii). $I\tau^*(\{\phi_N\}) = I\tau^*(L)$ i.e. $I\tau \subseteq I\tau^*$, for any intuitionistic ideal L_1 on X. In particular, we have for two topological intuitionistic ideals L_1, and L_2 on X, $L_1 \subseteq L_2 \Rightarrow I\tau^*(L_1) \subseteq I\tau^*(L_2)$.

Theorem 6.3

Let τ_1, τ_2 be two intuitionistic topologies on X. Then for any topological intuitionistic ideal L on X, $\tau_1 \leq \tau_2$ implies $IA^*(L,\tau_2) \subseteq IA^*(L,\tau_1)$, for every $A \in L$ then $I\tau^*{}_1 \subseteq I\tau^*{}_2$

Proof

Clear.

A basis $I\beta(L,\tau)$ for $I\tau^*(L)$ can be described as follows:

$I\beta(L,\tau)=\{A-B: A\in\tau, B\in L\}$. Then we have the following theorem

Theorem 6.4

$I\beta(L,\tau)=\{A-B: A\in\tau, B\in L\}$ Forms a basis for the generated IT of the IT (X,τ) with topological intuitionistic ideal L on X.

Proof

Straight forward.

The relationship between $I\tau$ and $I\tau^*(L)$ established throughout the following result which have an immediately proof .

Theorem 6.5

Let τ_1,τ_2 be two intuitionistic topologies on X. Then for any topological intuitionistic ideal L on X, $\tau_1\subseteq\tau_2$ implies $I\tau^*{}_1\subseteq I\tau^*{}_2$.

Theorem 6.6

Let (X,τ) be a ITS and L_1,L_2 be two intuitionistic ideals on X . Then for any intuitionistic set A in X, we have

i) $IA^*(L_1\cup L_2,\tau)=IA^*(L_1,I\tau^*(L_1))\cap IA^*(L_2,I\tau^*(L_2))$ ii) $I\tau^*(L_1\cup L_2)=(I\tau^*(L_1))^*(L_2)\cap(I\tau^*(L_2))^*(L_1)$

Proof

Let $p\notin(L_1\cup L_2,\tau)$, this means that there exists $U_p\in I(P)$ such that $A\cap U_p\in(L_1\cup L_2)$i.e. There exists $\ell_1\in L_1$ and $\ell_2\in L_2$ such that $A\cap U\in(\ell_1\vee\ell_2)$ because of the heredity of L_1, and assuming $\ell_1\wedge\ell_2=O_N$.Thus we have $(A\cap U)-\ell_1=\ell_2$ and $(A\cap U_p)-\ell_2=\ell_1$ therefore $(U-\ell_1)\cap A=\ell_2\in L_2$ and $(U-\ell_2)\cap A=\ell_1\in L_1$. Hence $p\notin IA^*(L_2,I\tau^*(L_1))$, or $P\notin IA^*(L_1,I\tau^*(L_2))$, because p must belong to either ℓ_1 or ℓ_2 but not to both. This gives $IA^*(L_1\cup L_2,\tau)\supseteq IA^*(L_1,I\tau^*(L_1))\cap IA^*(L_2,I\tau^*(L_2))$ .To show the second iIlusion, let us assume $P\notin IA^*(L_1,I\tau^*(L_2))$. This implies that there exist $U\in N(P)$ and $\ell_2\in L_2$ such that $(U_p-\ell_2)\cap A\in L_1$. By the heredity of L_2, if we assume that $\ell_2\subseteq A$ and define $\ell_1=(U-\ell_2)\cap A$ Then we have $A\cap U\in(\ell_1\cup\ell_2)\in L_1\cup L_2$. Thus, $IA^*(L_1\cup L_2,\tau)\subseteq IA^*(L_1,I\tau^*(L_1))\cap IA^*(L_2,I\tau^*(L_2))$ and similarly, we can get $IA^*(L_1\cup L_2,\tau)\subseteq IA^*(L_2,\tau^*(L_1))$. This gives the other iIlusion, which complete the proof.

Corollary 6.1

Let (X,τ) be a ITS with topological intuitionistic ideal L on X. Then

i) $IA^*(L,\tau)=IA^*(L,\tau^*)$ and $I\tau^*(L)=I(I\tau^*(L))^*(L)$

ii) $I\tau^*(L_1\cup L_2)=(I\tau^*(L_1))\cup(I\tau^*(L_2))$

Proof

Follows by applying the previous statement.

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